Lattice-Constrained Parametrizations of Form Factors for Semileptonic and Rare Radiative B Decays

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We describe the form factors for $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ and $B \to K^*\gamma$ decays with just two parameters and the two form factors for $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$ with three parameters. The parametrizations are constrained by lattice results and are consistent with heavy quark symmetry, kinematic constraints and light cone sum rule scaling relations.

We obtain a simple yet phenomenologically useful description of the form factors for semileptonic and rare radiative heavy-to-light meson decays for all q^2, the squared momentum transfer to the leptons or photon. Lattice calculations determine the form factors over a limited region at high q^2. We use model input to extend the results to q^2=0, seeking consistency with:

- kinematic constraints: $F_1(0) = F_0(0)$ and $T_1(0) = iT_2(0)$
- heavy quark symmetry (HQS)
- light cone sum rule (LCSR) scaling relations: all form factors scale like $M^{-3/2}$ as $M\to\infty$ at q^2=0, where M is the heavy meson mass
- dispersive constraints

The normalisation is fixed using lattice results. The outcome is a two parameter fit for $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ or $B \to K^*\gamma$ and a three parameter fit for $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$. More details can be found in [1].

The leading order HQS analysis shows that heavy-to-light $P\to P$ decay form factors are determined by two universal ("Isgur-Wise") functions, while $P\to V$ decays are governed by four more such functions (P and V denote pseudoscalar and vector mesons respectively). We adopt a model of Stech [2] which keeps just one universal function for $P\to P$ and one more for $P\to V$.

Lattice simulation details can be found in [3,4] with details of the chiral extrapolation for $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$ in [5]. All form factors are calculated for four values of the heavy quark mass around the charm mass and for a variety of q^2. In our previous work [3,4], the form factors were extrapolated at fixed four-velocity recoil, $\omega = v\cdot(p_{P,V}/m_{P,V})$, near the zero recoil point $\omega = 1$, using the heavy-quark scaling relations:

$$f\Theta M^{n_f/2} = \gamma_f \left(1 + \frac{\delta_f}{M} + \frac{\epsilon_f}{M^2} + \cdots\right)$$

where $n_f = -1, 1, -1, -1, 1, -1, -1, 1$ for $f = F_1, F_0, A_0, V, A_1, A_2, T_1, T_2$ and γ_f, δ_f and ϵ_f are fit parameters. Θ comes from leading logarithmic matching and is chosen to be 1 at the B mass. This procedure neglects the fact that for $M\to\infty$, HQS predicts $A_1 = 2iT_2$ and $V = 2T_1$ at fixed ω not too far from $q^2_{\rm max}$. We enforce this condition by performing a combined fit, at fixed ω, of the pairs (A_1, T_2) and (V, T_1) imposing the constraints: $\gamma_{A_1} = 2i\gamma_{T_2}$ and $\gamma_V = 2\gamma_{T_1}$. This guarantees that the extrapolated form factors are consistent with HQS in the infinite mass limit and reduces statistical errors by decreasing the number of parameters.

1. $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ AND $B \to K^*\gamma$ DECAYS

We use the freedom to adjust quark masses in lattice calculations and consider two situations for the light quark q into which the b decays:

Table 1
Form factor results for $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ and $B \to K^*\gamma$. For $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ the fit parameters are: $A_1(0) = 0.27(^{5}_{4})$, $M_1 = 7.0(^{12}_{6})\,\mathrm{GeV}$, $\chi^2/\mathrm{dof} = 24/20$. For $B \to K^*\gamma$: $A_1(0) = 0.29(^{4}_{3})$, $M_1 = 6.8(^{7}_{4})\,\mathrm{GeV}$, $\chi^2/\mathrm{dof} = 27/20$.

q^2	A_1	A_2	A_0	V	T_1	T_2
0	$0.27(^{5}_{4})$	$0.26(^{5}_{3})$	$0.30(^{6}_{4})$	$0.35(^{6}_{5})$	$0.16(^{2}_{1})$	
q^2_{max}	$0.46(^{2}_{1})$	$0.88(^{5}_{3})$	$1.80(^{9}_{5})$	$2.07(^{11}_{6})$	$0.90(^{5}_{4})$	$0.25(^{1}_{1})$

A $q{=}u$: matrix elements of $\bar{u}\,\sigma^{\mu\nu}(1{+}\gamma^5)b$ are unphysical but constrain $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$.

B $q{=}s$: $\bar{s}\gamma^{\mu}(1{-}\gamma^5)b$ is unphysical but constrains $B \to K^*\gamma$.

We complete the parametrization by specifying one of the form factors. To meet all our requirements, including the LCSR scaling condition at $q^2 = 0$, we choose

$$A_1(q^2) = \frac{A_1(0)}{1 - q^2/M_1^2} \qquad (1)$$

with free parameters $A_1(0)$ and M_1. This allows A_1, A_2 and T_2, which receive contributions from 1^+ resonances, to diverge at larger q^2 than the more singular V, A_0 and T_1. We also tried other parametrizations but all results below will use $A_1(q^2)$ in eq. (1). Figure 1 shows the fit for a final state with the mass of the K^*, and Table 1 gives results for the form factors.

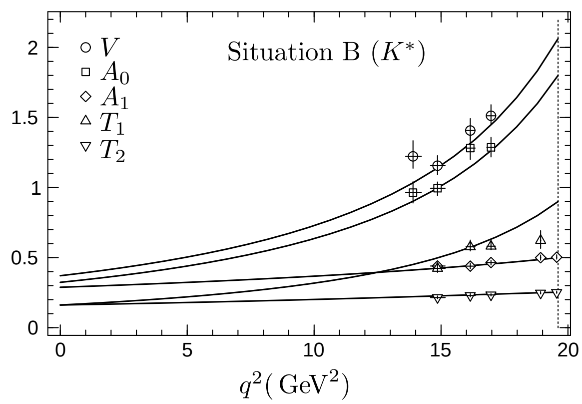

Figure 1. Fit to the lattice predictions for A_0, A_1, V, T_1 and T_2 for a K^* meson final state (Situation B) assuming a pole form for A_1. The dashed vertical line indicates q^2_{max}.

2. $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$ DECAYS

Stech's model makes $F_0(q^2_{\mathrm{max}})$ vanish in the chiral limit, contradicting our results and made unlikely by unitarity bounds [5]. Furthermore, the B^* which contributes a pole very close to q^2_{max} in F_1, induces the same singularity in F_0 in the model. This provokes a much stronger q^2 dependence for F_0 than seen in the lattice results or induced by the nearest 0^+ resonance. Therefore we restrict to polar-type q^2-dependences, consistent with the kinematical constraint, $F_1(0) = F_0(0)$, HQS and unitarity bounds. Our preferred model, consistent with LCSR scaling relations at $q^2 = 0$, is

$$F_1(q^2) = \frac{F(0)}{(1{-}q^2/m_1^2)^2}, \quad F_0(q^2) = \frac{F(0)}{(1{-}q^2/m_0^2)}.$$

The result of the fit is: $F(0) = 0.27(11)$, $m_1 = 5.79(58)\,\mathrm{GeV}$ and $m_0 = 6.1(15)\,\mathrm{GeV}$ with $\chi^2/\mathrm{dof} = 0.1/3$. All results below will be quoted using this pole/dipole model.

3. PHENOMENOLOGY

Using our fits we can calculate total rates and differential decay spectra in q^2 and lepton energy E for the decays $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ (Figure 2) and $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$. In Table 2 we give our results, illustrating the good agreement of our form factor values at q^2=0 with LCSR calculations, and compare rates and ratios for semileptonic decays. For $B \to K^*\gamma$ we evaluate the ratio $R_{K^*} = \Gamma(B \to K^*\gamma)/\Gamma(b \to s\gamma) = 16(^{4}_{3})\%$, to be compared with $(18 \pm 7)\%$ from experiment [6].

Table 2
Form factor values at $q^2 = 0$ with $B \to \pi, \rho$ semileptonic decay rates and ratios from this calculation and from light cone sum rules (LCSR). Decay rates are given in units of $|V_{ub}|^2\,\mathrm{ps}^{-1}$. $\Gamma_{\rho/\pi} \equiv \Gamma(\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l)/\Gamma(\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l)$ and $\Gamma_{L/T}$ denotes the ratio of rates to longitudinally and transversely polarised rho mesons in $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$. l denotes a massless lepton.

	$F_1(0)$	$A_1(0)$	$A_2(0)$	$V(0)$	$T_1(0)$	$\Gamma_{\pi l \bar{\nu}}$	$\Gamma_{\rho l \bar{\nu}}$	$\Gamma_{\rho/\pi}$	$\Gamma_{L/T}$	$\Gamma_{\pi\tau\bar{\nu}_\tau}$	$\Gamma_{\rho\tau\bar{\nu}_\tau}$
	0.27(11)	$0.27(^{5}_{4})$	$0.26(^{5}_{3})$	$0.35(^{6}_{5})$	$0.16(^{2}_{1})$	$8.5(^{33}_{14})$	$16.5(^{35}_{23})$	$1.9(^{9}_{7})$	$0.80(^{4}_{3})$	$5.8(^{18}_{4})$	$8.8(^{14}_{9})$
LCSR											
[7]		0.27(5)	0.28(5)	0.35(7)			13.5(40)	1.7(5)	0.52(8)		
[8]		0.24(4)		0.28(6)	0.16(3)						
[9]	0.24–0.29					8.7					
[10]					0.15(3)						

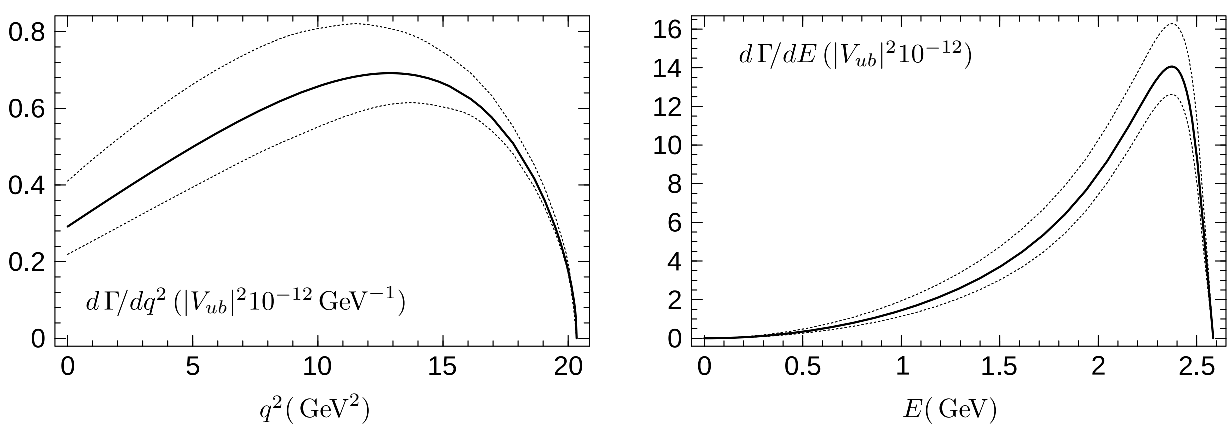

Figure 2. Differential decay spectra for $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ for massless leptons: (a) $d\Gamma/dq^2$ in units of $10^{-12}|V_{ub}|^2\,\mathrm{GeV}^{-1}$, (b) $d\Gamma/dE$ in units of $10^{-12}|V_{ub}|^2$. The dashed lines show the envelope of the 68% bootstrap errors computed separately for each value of q^2 or E respectively.

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